In His Name, the Most High

Linear Algebra

School of ECE, University of Tehran
acsl.ut.ac.ir/linearalgebra

Course Outline

Session #	Section	Detailed Topics	General Topics	Sources
1	1.1	Equilibrium Points & Energy of Dynamical Systems	Linear Algebra: A Big Picture	Miscellaneous
		Derivative of State Variables		
		Stable /Unstable Systems		
		Analog Computation		
		Complex vs. Real Matrices		
		Complex Numbers by Matrices		
		State Dependent Dynamical Systems		
2	2.1	Set and (Proper) Subsets	Set/Subset/Relation/Function	[5,7]
		Intersection / Union of Sets		
		Disjoint Sets		
		Difference of Sets		
		Index Sets		
		Equivalence Relations		
		Reflexivity, Symmetry and Transitivity of Relations		
	2.2	Domain and Codomain	Domain/Codomain/Range	
		Pre-Image and Image		
		Range		
		Equality of Functions (Transformation)		
3	3.1	Injection and Surjection	Injection/Surjection/Bijection	
		Surjection and its Relation to Range and Codomain		
		Bijection		
		Bijection and its Relation to Injection and Surjection		
		Restriction of a Function with Respect to a Set		
		Composition of Functions		
	3.2	Invertibility of a Function	Function Inversion	
		Invertibility vs. Bijection		
		Duality		
4	4.1	Algebraic Structures	Abstract Algebra	
		Group		
		Ring and Field		
		Binary Operation		
		Fundamental Theorem of Algebra		
		Algebraic Closeness		
	4.2	Sum of Vectors	Vectors	
		Multiplication by Scalars		

5	5.1	Commutativity /Associativity of Addition in Vector Spaces	Vector Spaces	[1,3]
		Zero, Symmetric, Identical and Inverse Elements		
		Field and Vector Elements		
		(Skew) Symmetric / (Skew) Hermitian Matrices		
	5.2	Subspace		
6	6.1	Linear Dependence / Independence	Vector Spaces (Contd.)	
		Replacement Theorem		
	6.2	Lagrange Interpolation	Bases and Dimension	
		Finite Vector Spaces		
		Flexible Structures (ODE → PDE)		
		Complex Vector Spaces		
		Generator Set		
		Venn Diagram		
7	7.1	Linear Transformation	Linear Transformations / Matrices	
		Reflection, Rotation, Projection, Identity, Derivative and Zero Transformations		
	7.2	Null Space, Kernel and Range	Null Space / Image	
		Rank and Nullity		
		Dimension Theorem		
8	8.1	Ordered Basis	Matrix Representation	
		Coordinate Vector		
		Vector Space of All Linear Transformations		
		Matrix Multiplication		
		Left Multiplication Transform		
9	9.1	Inverse of a Matrix	Invertibility / Isomorphism	
		Isomorphic Functions and Vector Spaces		
	9.2	Linear Operators	Coordinate Change	
		Change of Coordinates		
10	10.1	Elementary Matrix Operations	Solving Linear Equations	
		Gaussian Elimination		
		Triangular Factorization		
		LDU Decomposition		
		Pivoting		
11	11.1	Partial Pivoting		
		Round-off Error		
	11.2	Fundamental Theorem of Linear Algebra	Orthogonal Complement	
12	12.1	Dual basis	Dual Spaces Linear Functional	[1,4]
		Vector Mapping Using Dual Basis		
		Linear Functional		
		A Vector Space and its Double Dual		
		Annihilator of Subset / Subspace		
13	13.1	Solution Set	Homogeneous / General Solutions	[1]
		Consistent / Inconsistent System of Equations		
		Homogeneous / Nonhomogeneous Systems System of Equations		
		Echelon Form of a Matrix		
		General Solution		

14	14.1	Natural, Odd and Even Permutations	Permutation Group	[2]
	14.2	Basic Properties of Determinant	Determinant	
		Determinant Using Cofactors		
		Matrix Inverse Using Adjugate		
		Principle and Non-vanishing Minors		
		Determinant vs. Matrix Rank		
		Determinant vs. Volume		
15	15.1	Characteristic Polynomial	Diagonalizability	[1,2,3]
		Eigenvalues and Eigenvectors		
		Eigenspace		
		Test for Diagonalization		
16	16.1	Sum and Direct Sum of Subspaces	Direct Sum, Revisited	
		Diagonalization in Terms of Direct Sum		
	16.2	Column Sum	Disk Theorem	
		Eigenvalue Bound and Gerschgorin Disks		
	16.3	Convergence in Matrix Spaces	Matrix Limits / Markov Chains	
		Stochastic Matrices		
		Markov Chains and Process		
	16.4	Regular Transition Matrix	Regular Transition Matrices	
		Power of Regular Transition Matrix		
17	17.1	T-Invariant Subspaces	Invariant Subspaces	[1,3]
	17.2	Cayley-Hamilton Theorem		
19	19.1	Inner Product Spaces	Euclidean & Unitary Spaces	[1,3]
		Standard Inner Product		
		Frobenius Inner Product		
		Cauchy-Schwartz / Triangle Inequality		
		Orthogonal / Orthonormal Vectors		
		Norm and Distance		
		Orthonormal Basis		
		Gram-Schmidt Orthogonalization		
		Vector Projection		
	19.2	QR Decomposition Using Gram-Schmidt Procedure	QR Decomposition	
20	20.1	Data Fitting	Least Squares Approximation	
		Dimension Theorem		
		Minimal Solution		
21	21.1	Properties	Symmetric Matrices	
		Spectral Theorem		
		Pivots vs. Eigenvalues		
	21.2	Quadratic Functions Definite / Indefinite Matrices	Definite Matrices	
		Positive / Negative (Semi) Definite Matrix		
22	22.1	Schur Theorem	Normal / Self-Adjoint Operators	[1]
		Self-Adjoint vs. Hermitian Operators		
		Positive (Semi) Definite Operators		
		Unitary / Orthogonal Operators		
		Orthogonal Projection	Spectral Theorem	

23	23.1	Spectral Decomposition	Spectral Theorem (Contd.)	[1,6]
	23.2	Euclidean Space	Euclidean / Hermitian Spaces	
		Hermitian Space		
24	24.1	First / Second Order Approximation	Approximation and Extremal Points	[6,7]
		Stationary Points		
		Hessian Matrix		
		Algebraic / Geometric Multiplicity		
25	25.1	Singular Values / Vectors	Singular Value Decomposition	[3,7,8]
		Pseudo-Inverse		
		Image Compression via SVD		
26	26.1	Cholesky Factorization	Square Root of Positive Definite Matrices	[2,3]
		Diagonal Form of a Quadratic Function		
	26.2	Signature of a Symmetric Real Matrix	Congruent Transformation	
		Congruence Matrices		
	26.3	Generalized Eigenvalue Problem	Generalized Eigenvalue Problem	
27	27.1	Well-conditioned / Ill-conditioned Systems	Conditioning and Rayleigh Quotient	[1,3,7]
		Rayleigh Quotient and Condition Number		
		Euclidean Norms		
		Sensitivity Analysis for $Ax = b$		
	27.2	p-Norm	Vector Norm	
	27.3	Frobenius Norm	Matrix Norm	
		General Matrix Norm		
		Spectral Norm		
28	28.1	Nilpotent Matrix	Jordan Canonical Form	[1,3,5]
		Shift Matrix		
		Monic Polynomial		
		Minimal Polynomial		
		Defective Matrix		
		Generalized Eigenvector		
		Jordan Chains		
		Computation of Generalized Eigenvectors		
		Jordan Canonical Form and Differential Equations		
		Matrix Functions		

References

[1] Friedberg, S. H., Insel, A. J. and Spence, L. E., *Linear algebra*, 4th Edition, Prentice Hall, 2002.

[2] Banerjee, S., and Anindya, R., *Linear algebra and matrix analysis for statistics*. CRC Press, 2014.

[3] Strang, G., *Introduction to Linear Algebra*, 5th Edition, Wellesley - Cambridge Press, 2016.

[4] Lipschutz, S., and Lipson, M. L., *Linear Algebra: Schaum's Outlines*. McGraw-Hill, 2009.

[5] Bernstein, D. S., *Matrix mathematics: theory, facts, and formulas*. Princeton university press, 2009.

[6] Gallier, J., *Fundamentals of linear algebra and optimization*. University of Pennsylvania 2014.

[7] Chen, C. P., *Linear Algebra*. Lecture Notes.

[8] Burl, J. B., *Linear Optimal Control: H_2 and H_∞ Methods,* Addison-Wesley Longman Publishing Co. Inc., 1998.